Chapter 2

Waves and Plane Sources

Not all wave motion results in sound. In this chapter, waves and types of wave motion are discussed. The sound generated by plane surfaces is also discussed. Although a plane source is, in reality, a distribution of point sources, several important aspects of sound generation can be elucidated simply without the geometric complexity inherent in spherical coordinates.

2.1 Wave Motion

2.1.1 Definition of a Wave

A wave has variable displacement in either time or space. There are three categories of waves of interest:

1. those that are fixed relative to a medium, but move relative to an observer.
2. those that move relative to a medium but are fixed in time with respect to an observer.
3. those that move relative to a medium and move relative to an observer.

Examples of the first category are washboards on a road; the waves have spatial dependence but no time dependence. If a vehicle travels over the washboards, is sound necessarily created? Examples of the second category are "standing" waves, such as those found behind a rock in a water stream or those behind a mountain in an air stream (lenticular clouds). The waves have time dependence relative to the medium but only spatial dependence relative to a fixed observer. Do such waves make sound? No. If an observer travels through lenticular clouds on an aircraft, will sound necessarily be created? That is a different story; both the moving observer and a fixed observer on the ground will hear sound. The third category has two parts. The words "*traveling waves*" are used to describe waves that have both spatial and time dependence, and appear to move relative to an observer. The words "*standing waves*" are also used to describe waves *traveling* between two reflecting surfaces. Such waves have both spatial and time dependence, but *appear* not to be moving relative to an observer. Will the observer hear sound? Yes. The true traveling wave case is the one most commonly encountered.

Emphasis in this monograph is on the sound generation process. A fluid or solid must have *elastic* properties so that waves can exist in it. The waves will have a defined speed of propagation relative to the medium, a magnitude, and a direction of motion. Pressure is a measurable aspect of a wave in a compressible medium, so it is generally emphasized over the other aspects (mostly because it can be measured rather easily). If those waves generate a time dependent pressure fluctuation at an observer's ear, it will be interpreted generally as "sound", but it may not be. As will be shown in later chapters, there are time dependent pressure fluctuations that are not sound. Although both types of pressure fluctuations are discussed, the emphasis is on sound that propagates to distant locations.

Key Points: The correlation between the word "waves" and the word "sound" is not always one. Also, the correlation between pressure fluctuations that are heard and the sound generated by a source is not exact.

2.1.2 Motion of Plane Waves

Consider a wave moving in a positive (or negative) direction in space-time; the wave may have an (almost) arbitrary form as shown in the first of Eqs. 2.1. The wave shape at position zero and time zero has the same shape at distance **r** or at the delayed (or advanced) time **t.** As long as the wave equation is valid {Appendix D} the time history at a remote location will be the same as that experienced near the source. For other geometries the time history is similar but the magnitudes are different. The wave form is applicable to both deterministic and random waves. The variable **c** has the dimensions of L/T and is interpreted to be a speed of propagation.

The second equation of Eqs. 2.1 may be used to express the amplitude **p** as a function of time and space when the wave is sinusoidal (one frequency). The exponential function is a compact way of showing this dependence {Appendix B}. The most commonly used form is the term on the furthest right and will be the form used in later chapters. The wave number **k** is based on the relationship between the frequency **f** and wavelength **λ** for a wave moving at speed **c,** as shown in the third expression. The wave number is **not** a number; it has the dimensions of inverse length: L^{-1}. Note that the wavelength is defined simply as a rotation of 2π (full circle). Note that the term **kr** is the distance in wavelengths, a **dimensionless** number.

$$
\begin{aligned}
&f(0,0) = f\left(ct - r\right) = f\left(t - \frac{r}{c}\right) = f\left(ct + r\right) = f\left(t + \frac{r}{c}\right) \\
&p\left(r,t\right) = p_1 e^{i\omega t}\Big|_{r=0} = p_1 e^{i\omega\left(t - \frac{r}{c}\right)}\Bigg|_{r>0} = p_1 e^{i(\omega t - kr)}\Big|_{r>0} \qquad (2.1) \\
&k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}
\end{aligned}
$$

Key Point: Waves with arbitrary shape that move in space-time without change of shape have a simple descriptor when they satisfy the wave equation. Mathematically, the function must not be discontinuous. Sound waves in air are not discontinuous. Ocean waves approaching a beach become discontinuous and curl over. For single frequency waves, specific relationships can be expressed.

2.1.3 The Elementary Plane Wave Equation

What equation of motion does the first of Eqs. 2.1 satisfy? Take the double space and time differentials for both the positive and negative functions; the result is shown in the first equation of Eqs. 2.2. The second equation results by equating the two double differentials, with

$$
\begin{aligned}
&\frac{\partial f^2}{\partial r^2} = f'', \frac{\partial f^2}{\partial t^2} = c^2 f'' \qquad (2.2) \\
&\frac{\partial f^2}{\partial r^2} - \frac{1}{c^2}\frac{\partial f^2}{\partial t^2} = 0
\end{aligned}
$$

an arbitrary constant set to zero. This must be the equation for the propagation of waves with unchanging shape. It is called the *homogenous* wave equation because that constant is zero. If the term on the right is non-zero, it implies a forcing function (a sound source). The mathematical complexity inherent in including such a function can be bypassed by studying only the space beyond the source; very little is lost in doing so. Since the variable **r** is simply a distance, a Cartesian coordinate system can be used to generalize the form of the equation. The first of Eqs. 2.3 below shows each of the Cartesian dimensions explicitly. The second uses vector notation. The upside down triangle is called the *Laplacian operator*, named after Pierre-Simon, marquis de Laplace (1749-1827) a French astronomer and mathematician. The square symbol encompasses the entire equation and is called the *D'Alembertian* or *wave operator* and is named after Jean le Rond D'Alembert (1717-1793). Note that if a fourth spatial dimension is defined as x_4 = ict, then the wave operator is merely a four dimensional Laplace equation. Did D'Alembert precede Einstein in defining space-time? The third equation is the wave equation in Cartesian *tensor* notation where **i** varies from 1 to 3 (by convention, the summation sign is implicit but omitted). The dates when these prodigious minds lived shows how early many of our modern concepts evolved.

It can be shown that the solution of the wave equation is correct for any ***continuous*** wave shape.

$$\frac{\partial f^2}{\partial x^2}+\frac{\partial f^2}{\partial y^2}+\frac{\partial f^2}{\partial y^2}-\frac{1}{c^2}\frac{\partial f}{\partial t^2}=0$$

$$\nabla^2 f-\frac{1}{c^2}\frac{\partial f}{\partial t^2}=\Box^2 f=0 \qquad (2.3)$$

$$\frac{\partial f^2}{\partial x_i^2}-\frac{1}{c^2}\frac{\partial f}{\partial t^2}=0$$

Key Points: The wave equation can be expressed in several symbolic and geometric forms, and the solution for it can be expressed in general terms. Since the present subject is waves in air, the applicability of this equation to actual situations depends on a number of approximations to the equations of fluid motion {D.2}.

2.2 Wave Interactions

A number of variations to traveling waves occur and are described below. Trigonometric forms are used here in lieu of complex forms to show the physics more clearly. The amplitudes **A** should be considered as peak values.

2.2.1 Traveling vs. Standing Single Frequency Waves

A traveling wave is the type discussed above. Standing waves that vary in time but do not appear to travel can be constructed from two equal, but opposing, traveling waves.

$$f\left(r,t\right)=\sum_{m=1}^{n}A_m\left[\cos(\omega_m t-k_m r)+\cos(\omega_m t+k_m r)\right]=2\sum_{m=1}^{n}A_m\sin\omega_m t\sin k_m r \qquad (2.4)$$

$$If\ \sin k_m r=\sin m\pi=0, then\frac{\lambda}{2}=\frac{r}{m}$$

The time variations and spatial variations are separated by use of a standard trigonometric identity. Although the wave amplitude clearly varies with time, there are locations for each frequency in which the amplitude is zero, called the *node points*. This result is the exception to the standing waves described in {2.1.1}; it is actually composed of two traveling waves with a

specific phase relationship, typically caused by reflections from two facing surfaces. One interesting sidelight is that the sound intensity is zero (the two intensities cancel) although the sound pressure is finite, so it is possible to hear a sound with **zero intensity**!
Key Points: In the presence of a reflecting surface, measurements of sound pressure may not be representative of the undisturbed sound field. A classic example is a person holding a sound level meter between themselves and the sound source. The measurement of sound pressure does not always imply the correct sound intensity and thus the sound power.

2.2.2 Amplitude Modulated Single Frequency Waves

AM radio uses amplitude modulated electrical signals and the same process applies to sound. Consider two signals, one the carrier and one the modulation (information) signal which is less than the carrier frequency. Without modulation (ω_m=0) the output signal **P** in Eq. 2.5 oscillates with amplitude $\mathbf{A_0}$ at the *carrier frequency* (ω_c). With modulation, the amplitude of **A** varies with time. When standard trigonometric identities are used, three signals result, one at the carrier frequency with no modulation and two *sidebands* of half amplitude. The information is carried in the sidebands only. Note that since the two frequencies are arbitrary, they do not

$$\begin{aligned}
P &= A\cos(\omega_c t) \\
A &= A_0 + A_1\cos(\omega_m t) \\
P &= A_0\cos(\omega_c t) + A_1\cos(\omega_m t)\cos(\omega_c t) \\
P &= A_0\cos(\omega_c t) + \frac{A_1}{2}\left[\cos[(\omega_c+\omega_m)t] + \cos[(\omega_c-\omega_m)t]\right]
\end{aligned} \tag{2.5}$$

necessarily have a harmonic relationship. To carry speech information, the modulation frequency must be expanded to a frequency spectrum, so the two sideband lines shown in Figure 2-1 must be broadened to include a band of frequencies. If $\mathbf{A_1}$ is small, the amplitude envelope of the signal will vary only slightly. When $A_1/2 = A_0$, the amplitude envelop will go to zero at the modulation frequency rate. If $A_1/2 > A_0$, the modulation envelop will cross over, creating additional zeros. These latter conditions are avoided in AM radio.

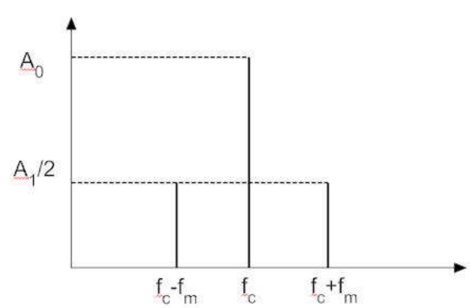

Fig. 2-1. Amplitude modulation spectrum.

Key Point: The importance here is that if a machine sound is modulated in a periodic way, e.g., RPM changes, a frequency below the main frequency (*sub-harmonic)* is generated. This modulation might be attributed erroneously to non-linear motion, an additional source, or an additional path.

2.2.3 Phase Modulated Single Frequency Waves

Phase modulation is less common. For this example single frequency modulated phase

$$
\begin{aligned}
P &= A_0 \cos(\omega_c t + \theta) \\
\theta &= A_1 \cos(\omega_m t) \\
P &= A_0 \cos(\omega_c t + A_1 \cos(\omega_m t)) \\
P &= A_0 \left[\cos(\omega_c t)\cos(A_1 \cos(\omega_m t)) - \sin(\omega_c t)\sin(A_1 \cos(\omega_m t))\right]
\end{aligned} \tag{2.6}
$$

shift will be used. There is no need for a fixed phase as it merely shifts the initial value of **P**.

Case 1: $A_1 << 1$

For this case there are only two significant side bands similar to amplitude modulation.

Case 2: $A_1 \le 1$

This case is more complex and can be handled by series expansions. The exact result is best presented in software, since the expansion terms get quite long. Calculations for A_1=1 yield the spectrum shown in Figure 2-2 which has a multiplicity of sidebands whose relative levels are

$$\cos(A_1 \cos(\omega_m t)) \approx 1$$

$$\sin(A_1 \cos(\omega_m t)) \approx A_1 \cos(\omega_m t)$$

$$P = A_0 \cos(\omega_c t) - \frac{A_1}{2}\left[\sin[(\omega_c + \omega_m)t] + \sin[(\omega_c - \omega_m)t]\right]$$

given in Table 2-1 below. They drop relatively sharply.

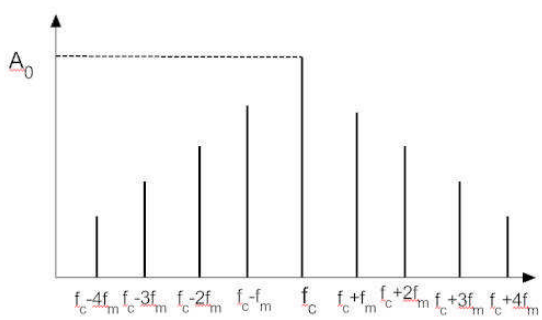

Fig. 2-2 Phase modulation spectrum.

N	0	1	2	3	4
dB	0	-4.7	-16.3	-31.6	-49

Table 2-1. Amplitude of harmonics for a frequency modulated wave.

2.2.4 Frequency Modulated Single Frequency Waves

$$
\begin{aligned}
&P = A_0 \cos(\omega t) \\
&\omega = \omega_c + A_1 \cos(\omega_m t) \\
&P = A_0 \cos((\omega_c + A_1 \cos \omega_m t)t) \qquad (2.7) \\
&P = A_0[\cos\left[A_1 \sin\left(\omega_m t\right)\right]\cos(\omega_c t) - \sin\left[A_1 \sin\left(\omega_m t\right)\right]\sin(\omega_c t)] \\
&P_n = J_{2n}\left(A_1\right)\left[\cos((\omega_c + 2n\omega_m)t) + \cos((\omega_c - 2n\omega_m)t)\right]
\end{aligned}
$$

FM radio uses frequency modulated electrical signals and the same concept applies to sound. Consider two signals, one the carrier and one the modulation (information) signal which is *less* than the carrier frequency.

To expand these functions further requires the use of Bessel function of the first order. The latter equation is representative of the nth sideband whose amplitude is expressed with the Bessel function which is oscillatory itself, so the sidebands do not decay monotonically as with the earlier types of modulations.

2.2.5 Beats

Beats create a form of modulation but are created by two signals of *slightly different* frequency, not the modulation of one signal. The sound from small twin engine propeller planes is an example of beats where the synchronizer attempts to match RPM, but never perfectly.

The two frequencies are subscripted **c** and **m** as in the previous sections. If they are greatly different, no beat is detected, but if they differ by small amounts, beats are heard. For this case we define ω_d which is the difference between the frequencies. If that difference is small

$$
\begin{aligned}
&P = A_0\left[\cos(\omega_c t) + \cos(\omega_m t)\right] \\
&P = 2A_0 \cos\left(\frac{(\omega_c - \omega_m)}{2} t\right)\cos\left(\frac{(\omega_c + \omega_m)}{2} t\right) \qquad (2.8) \\
&P = 2A_0 \cos\left(\frac{\omega_d}{2} t\right)\cos\left(\left(\omega_c - \frac{\omega_d}{2}\right)t\right)
\end{aligned}
$$

the additive factor in the second term of the third equation in Eqs. 2.8 may be neglected and we now have a situation similar to amplitude modulation where the modulation frequency is now the difference frequency.

Key Points: Apparent modulation of one sound source may be caused by two nearly synchronous sources. The identification is that the modulation frequency is considerably below that of the main source.

2.2.6 Finite Amplitude Waves

The speed of sound **c** is based on isentropic

$$c = c_m + u = \sqrt{\gamma R\left(T_0 + T\right)} + u$$
$$c_m = \sqrt{\gamma R T_0}\sqrt{1+\frac{T}{T_0}} = c_0\left(1+\frac{T}{2T_0} - .....\right) \approx c_0\left(1+\frac{\gamma-1}{2}s\right)$$
$$u = c_0 s$$
$$c \approx c_0\left(1+\frac{\gamma+1}{2}s\right) \quad (2.9)$$

conditions and for a perfect gas, i.e., small amplitudes {D.3}. A simple approach for larger amplitudes is as follows.

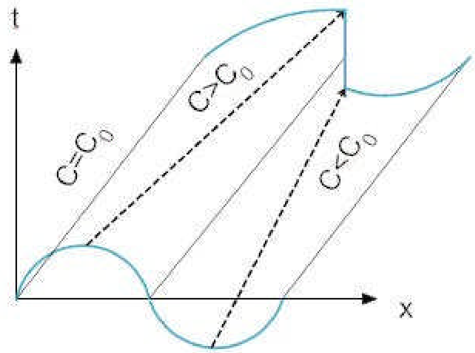

Fig. 2-3 Wave Distortion

The wave speed is the sum of the propagating wave in the medium plus the medium motion. **T** is the temperature fluctuation and the square root expansion yields a first order expression in terms of condensation (See Eq. 2.10). When the medium motion is added the final relationship is as shown. At the peak (s is positive) the wave travels faster than the trough (s is negative). As a result the wave front steepens. The condensation **s** is quite small at normal level sounds. Noticeable steepening occurs around levels of 170 dB, a pressure of around 1 psi, so use of the small amplitude sound speed is warranted for lesser levels. With time, the wave front in air steepens to a *shock wave*. When flow exits from a nozzle at 1.893 times (or more) the atmospheric pressure, the flow is critical and the flow goes supersonic {4.10.6} and shock waves are formed. If this were an oscillatory pressure, it would equate to a sound level of 200 dB. Similarly, for aircraft moving faster than the speed of sound, the wavelets created steepen rapidly to create shock waves.

Key Point: The amplitude of most sound waves is sufficiently small that the wave shape remains constant so the spectrum contour of the sound does not necessarily change with distance.

2.3 Plane Wave Sources

The case of a large flat rigid surface that oscillates perpendicular to itself is given here. Motion in the plane of the surface is deferred until {2.3.7.1} where viscosity plays a role not included in the standard wave equation (Eq. 2.3).

2.3.1 The Equations

The equations for the physical variables associated with waves created by the perpendicular motion of a plane surface in a normal fluid are given in Eqs. 2.10. The subscript zero refers to static values. See the development in {A.3} for details of these variables. We see that by use of a solution for velocity potential, the various physical variables can be determined easily. Since interest is in sound generation, only the outgoing waves are discussed. Note that the pressure **p**, density **s**, and temperature **T** variables are the changes from the *non-zero* static

$$\frac{\partial^2\phi}{\partial x^2}-\frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2}=0$$
$$c_0^2=\frac{\gamma p_0}{\rho_0}=\gamma RT_0$$
$$p(x,t)=\rho_0\frac{\partial\phi}{\partial t} \tag{2.10}$$
$$u\left(x,t\right)=-\frac{\partial\phi}{\partial x}$$
$$s(x,t)=\frac{1}{c_0^2}\frac{\partial\phi}{\partial t}=\frac{\rho-\rho_0}{\rho_0}$$
$$T(x,t)=\frac{1}{R}\frac{\gamma-1}{\gamma}\frac{\partial\phi}{\partial t}$$

values while the motion variables are changes from *zero* values.

2.3.2 Force Generated Source

Consider the case of a surface with area **A** that is large with respect to a wavelength and having a sinusoidal force **F** applied to it. The magnitude of the force per unit area would be $\mathbf{p_1}$. The velocity potential is first developed from the pressure and then the other variables are

$$p(x,t)=p_1e^{i(\omega t-kx)}$$
$$\phi\left(x,t\right)=\frac{p_1}{i\omega\rho_0}e^{i(\omega t-kx)}$$
$$u\left(x,t\right)=\frac{p_1}{Z_0}e^{i(\omega t-kx)} \tag{2.11}$$
$$s(x,t)=\frac{p_1}{Z_0c_0}e^{i(\omega t-kx)}$$
$$T(x,t)=\frac{1}{R}\frac{\gamma-1}{\gamma}\frac{p_1}{\rho_0}e^{i(\omega t-kx)}$$

derived from it, all are expressed in terms of surface pressure in Eqs. 2.11.

First note that all the properties vary with the applied force, but only the displacement and acceleration vary with frequency. For an applied force of 0.01 pound over an area of one square foot we get

Frequency, Hz	Pressure Change lb_f/in	Acceleration ft/sec^2	Velocity ft/sec	Displacement Inches	Density change lb_m/in^3	Temperature change °F	L_p dB
500	0.0000694	12.16	0.00387	0.0000148	0.000000257	0.000715	87.6
2000	0.0000694	48.6	0.00387	0.00000370	0.000000257	0.000715	87.6

Table 2-2. Value of physical variables at two frequencies for a constant force plane wave.

Note that the density and temperature are in phase, so the compression of the medium results in a temperature rise. This is based on the process being adiabatic, and is reasonable since it is unlikely that heat transfer can occur for such a fast process.

The Sound Pressure Level is shown below. The magnitude of $\mathbf{p_1}$ should be interpreted as the root-mean-square (rms) value, *not the peak value* {B.4}.

$$L_p = 10\log_{10}\left[\frac{p_1^2}{p_R^2}\right] = 20\log_{10}\left[\frac{p_1}{p_R}\right]$$

Key Point: It does not take much force to generate a high sound pressure level. Note that the pressure and velocity are in phase, so all the motion is resistive, i.e., all the work applied to the medium at the boundary is transferred to the medium as sound.

2.3.3 Displacement Generated Source

Consider the same situation as in the previous section except that the sinusoidal force is replaced by a sinusoidal displacement of fixed amplitude.

$$\begin{aligned}
\varepsilon(x,t) &= \varepsilon_1 e^{i(\omega t - kx)} \\
\phi(x,t) &= \varepsilon_1 c_0 e^{i(\omega t - kx)} \\
p(x,t) &= i\omega\varepsilon_1 Z_0 e^{i(\omega t - kx)} \\
u(x,t) &= i\omega\varepsilon_1 e^{i(\omega t - kx)} \\
s(x,t) &= ik\varepsilon_1 e^{i(\omega t - kx)} \\
T(x,t) &= i\omega\varepsilon_1 \frac{1}{R}\frac{\gamma - 1}{\gamma} e^{i(\omega t - kx)} \\
L_p &= 20\log_{10}\left[\frac{\omega\varepsilon_1 Z_0}{p_R}\right]
\end{aligned} \qquad (2.12)$$

In this case many of the variables are now dependent on frequency unlike the previous case. There is a displacement problem; the velocity of the surface must match that of the fluid at the boundary **ξ** which is not zero. A series expansion yields the relationship on the left.

$$u(\varepsilon,t) = Ue^{i(\omega t - k\varepsilon)} \approx Ue^{i\omega t}\left[1 - ik\varepsilon + \frac{k^2\varepsilon^2}{2!} - .....\right] \qquad k|\varepsilon| = \frac{\omega}{c_0}|\varepsilon| = \frac{|u|}{c_0}$$

Looking at the form of the velocity in Eqs. 2.12, we see that the magnitude of the succeeding terms in the expansion are in powers of the local Mach number (as seen in the expression on the right). Thus the Mach number of the wall motion must be small in order to permit the approximation that the interface is at x=0. Another, and more satisfying, view is that the displacement must be small compared to the wavelength. Once again the pressure, velocity, density and temperature are all in phase, so all motion dissipates the power by radiation and nothing is stored temporarily. Note that if the area of the plane is finite, say **A**, it can be considered to be a volumetric flow source **uA**.

Key Point: The Mach number of a vibrating surface must be small if the frequency spectrum of the sound field is to be the same as that of the source.

2.3.4 Thermally Generated Source

Normally, one is not concerned with sinusoidal oscillations of temperature as a source of sound; the magnitude of temperature changes associated with normal sound pressure levels are small. A large and abrupt temperature change can result in significant levels. In this case, the temperature is given; the resultant relations are shown on the right. The Sound Pressure Level is

$$T(x,t) = T_1 e^{i(\omega t - kr)}$$
$$\phi(x,t) = \frac{\alpha T_1}{i\omega} e^{i(\omega t - kr)}$$
$$p(x,t) = \alpha \rho_0 T_1 e^{i(\omega t - kr)}$$
$$u(x,t) = \frac{\alpha T_1}{c_0} e^{i(\omega t - kr)}$$
$$s(x,t) = \frac{\alpha T_1}{c_0^2} e^{i(\omega t - kr)}$$
$$\alpha = \frac{\gamma R}{(\gamma - 1)}$$

$$L_p = 10\log_{10}\left[\frac{\alpha^2 \rho_0^2 T_1^2}{p_R^2}\right] = 20\log_{10}\left[\frac{a p_0 T_1}{p_R}\right] \qquad (2.13)$$

T_1 should be interpreted as the rms value, *not the peak value* {B.3}.

An unusual situation is the periodic, diurnal temperature variation through the wall of a building. Is sound created by this process? According to the equations, the answer would be yes, but the frequency would be at 1.1 x 10^{-5} Hz, well out of the human hearing envelope! This is an example of working beyond the bounds of validity. The equation of state {D.3} requires small amplitude and *rapid* temperature changes which these are not, so no is the correct answer. A more realistic application is a current pulse through a thin metallic plate of finite resistance. The power absorbed can be considered a step function, resulting in a rapid temperature rise and thus sound.

Key Point: Temperature changes as sound generating mechanisms in plane surfaces are rare. There are many cases of thermally generated sources in other geometries, such as lightning, corona noise, and combustion {3.11.1} and consideration should be given to it.

2.3.5 Acoustic Loading of a Vibrating Surface

When vibration of a plate is considered, acoustical aspects are seldom addressed. Is acoustical loading a significant factor? Consider the simple mass, spring, resistor arrangement shown in Figure 2-4. Many books show the methods of analysis; the relevant equations are shown below.

$$M\ddot{\xi} + (R + Z_0 A)\dot{\xi} + K\xi = 0$$
$$\xi = Be^{\alpha t}$$
$$\omega_n^2 = \frac{K}{M}, \zeta = \frac{(R + Z_0 A)}{2M\omega_n} \qquad (2.14)$$
$$\omega_d = \omega_n\sqrt{1-\zeta^2}, \alpha = \zeta\omega_n \pm i\omega_d$$
$$\xi = Be^{-\zeta\omega_n t}\cos(\omega_d t + \theta)$$

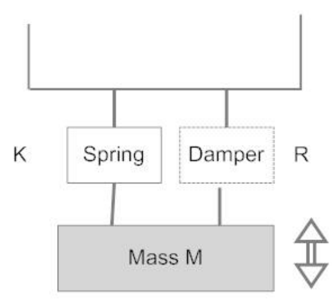

Fig. 2-4 Mass-Spring

The symbol ξ is the displacement, and $\boldsymbol{\omega_n}$ is the undamped natural frequency. The damped natural frequency $\boldsymbol{\omega_d}$ depends on both the internal damping **R** and the radiation damping $\mathbf{Z_0A}$. The acoustical loading appears in the resistance term; since it is per unit area the mass area needs to be included. If the damping factor is less than one, the last equation applies. What is the effect of acoustic loading on the resonance frequency and on the rate at which the amplitude decays? The geometry of the mass is quite important. If the mass is thick with a small surface area the acoustic loading is small. If the mass is a thin plate, the story is different. Figure 2-5 shows the amplitude decay as a function of time with, and without, acoustic damping. The data were: M=17 lb_m, R=10 lb_f-sec/ft, K=30 lb_f/in. The mass was 1.75 inches thick and 9.75 inches square. The damped natural frequency was 24.3 Hz. without acoustic loading and 23.6 Hz with it.

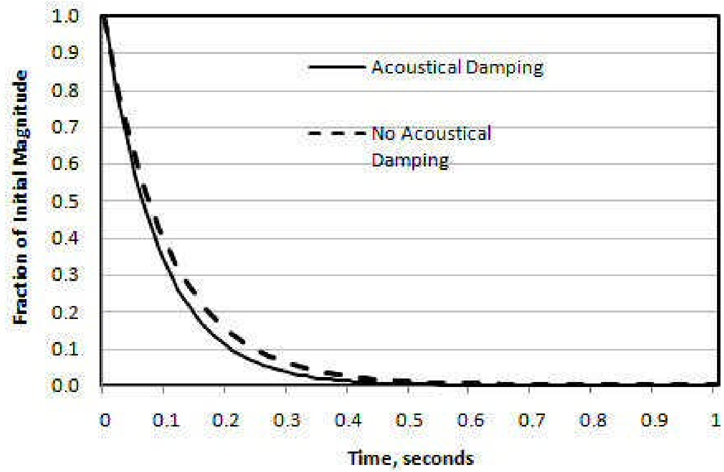

Fig. 2-5 Acoustic damping of a vibrating plate.

There are several other aspects related to control of sound emission from vibrating surfaces. Figure 2-6 shows the velocity response of a mechanical resonance system with damping as a function of frequency being driven by a constant force. Every text on mechanical vibration has equivalent graphs, but often attempts to reduce sound emission from such a system make poor use of it. The response on the right side of the graph is called the *mass controlled* region while that on the left side is called *the stiffness controlled* region. Between them is the *damping controlled* region. The key, of course, is to know what the natural (damped resonance) frequency is with respect to the frequency range of human hearing.

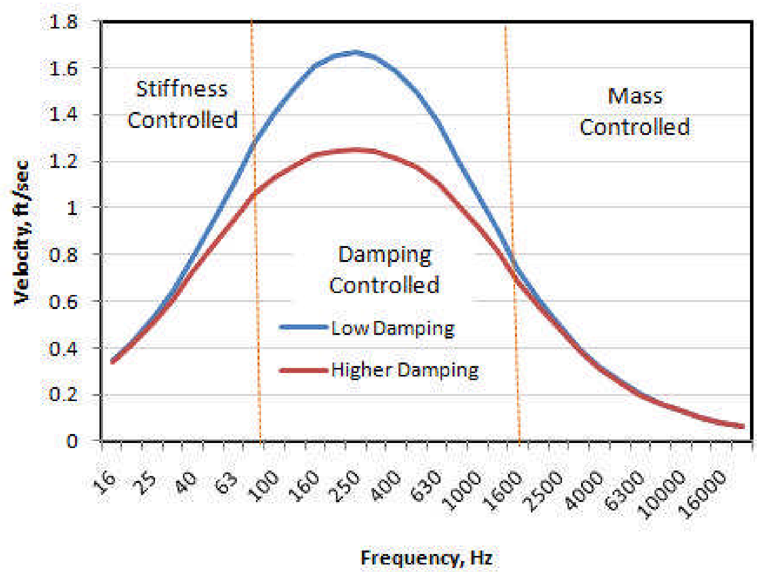

Fig. 2-6 Response of a resonant mechanical system to a constant force.

If the resonant frequency is low, it is beneficial to add *limp*, not stiff, mass to drive it to a lower frequency. Stiffness has been added for this situation in the mistaken assumption that it will reduce sound emission because it reduces displacement motion.

The role of damping in the graph is clear. It is highly beneficial to reduce resonance response, but only if that range of frequencies is relevant. Damping has been added to plates in which the resonant frequency is so low the only function of the material was to add a little mass and a lot of cost. The same is true in the stiffness controlled region. For plates with multiple resonances, damping is very beneficial.

What basic damping mechanisms are there? The parentheses show the proportionality of the damping forces.

- Fluid viscous $\left(\dot{\xi}\right)$
- Fluid compressible pressure (sound) $\left(\dot{\xi}\right)$
- Fluid laminar incompressible pressure $\left(\dot{\xi}\right)$
- Fluid turbulent incompressible pressure $\left(\dot{\xi}^2\right)$
- Solid Hysteresis $\left(\xi\right)$
- Solid Coulomb (Dry friction)

Key Points: Large radiating areas play a significant role in the sound radiated from a surface. Care must be taken when choosing the methods of velocity reduction on resonant structures.

2.3.7 Influence of Viscosity

The wave equation does not contain viscous terms. Does that have significant influence on results?

2.3.7.1 Motion in the Plane of a Flat Surface

When a plane surface moves in the same direction as the plane, the fluid is not compressed and viscous forces control. Eq. 2.15 shows the fluid velocity **v** in the direction of the plane **y** as a function of the distance **x** perpendicular to the plane for a single frequency. The medium responds to the motion of the plane by propagating a damped shear wave perpendicular to the plane. The damping constant is proportional to the frequency and inversely to the viscosity.

$$v\left(x,y,t\right)=v_0 e^{-\alpha x}\cos\left(\omega t-ky\right) \tag{2.15}$$

Key Points: Motion in the plane of a surface creates only transverse or shear waves and will not create sound. Exponentially damped shear waves are propagated into the medium. The characteristic speed is that of the surface motion. The characteristic length is the wavelength of the motion.

2.3.7.2 Motion Perpendicular to the Plane of a Flat Surface

When the surface moves perpendicular to itself with viscous terms included in the governing equations, the solution for a single frequency is as shown in Eqs. 2.16.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2}$$

$$u(x,t) = Ae^{i(\omega t - kx)} \tag{2.16}$$

$$k = \frac{\omega}{c_0}\left[1 + \frac{i\nu\omega}{c_0^2}\right]^{-1/2} \cong \frac{\omega}{c_0} + \frac{i\nu\omega^2}{2c_o^3}$$

$$u(x,t) = Ae^{-\frac{\nu\omega^2}{c_0^3}x}\cos\left[\omega\left(t - \frac{x}{c_0}\right)\right] = Ae^{-\frac{4\pi^2}{\mathrm{Re}_\nu}\frac{x}{\lambda}}\cos\left[\omega\left(t - \frac{x}{c_0}\right)\right]$$

$$\mathrm{Re}_\nu = \frac{c_0\lambda}{\nu}$$

The expansion of the wave number **k** is based on the second term being small. Viscous effects do attenuate sound. The attenuation increases rapidly with frequency. How much? The attenuation coefficient can be likened to an acoustical version of the Reynolds number. In air, for a frequency of 1000 Hz, the attenuation is 4.5×10^{-6}/foot. The neglect of viscous effects for most frequencies of sound in air is warranted.

Key Points: Motion perpendicular to a surface creates longitudinal (compression) waves and will create sound. Viscosity causes exponential decay of the sound amplitude, but the attenuation coefficient is sufficiently small that it can be neglected in the hearing range of humans.

2.3.8 Limitations of Plane Wave Analysis

Can unbounded plane waves actually exist? In the above analyses, the source is presumed to be an infinite plane, all parts of which are fully coherent. Although the model provides much insight, it is not physically realizable. The first clue, of course, is that the sound level does not decrease with distance! The closest approximation to the real world would be a finite plane surface whose dimensions are longer than several wavelengths. It must also be of significant depth in wavelengths to avoid communication with the back side. If the normal frequency range of hearing is considered, this would imply a large and very stiff structure. Consequently, the best approximation to a plane source is to be in the plane wave region, the triangular region shown in Figure 2-7. This limited region has some value in that useful information about *local* motion of a complicated source can be obtained.

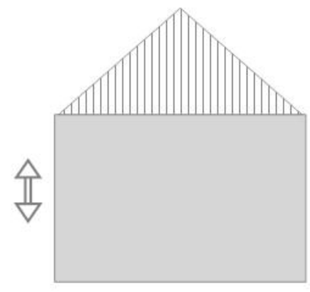

Fig. 2-7. Plane wave region of a large source.

Of course bounded plane waves can exist, such as the propagation of sound in a duct. However, when this geometry exists, the presence of boundaries allows for other modes of propagation, called *cross-modes*. Depending on the shape of the boundary (e.g., a rectangular or cylindrical duct), these modes can take on several forms and it is necessary to separate them from the plane wave mode.

2.4 Body Wave Types

It is possible to class mechanical waves into two types; body and surface. Both types have the ability to result in sound.

2.4.1 Longitudinal (Compression) Waves

These waves result in the compression of the medium (density) changes. The main characteristic is that the material motion (either solid or fluid) is in the same direction as that of the wave. The wave is associated with pressure forces that create the motion. Each of the basic sound sources creates this type of wave to propagate the source energy to distant locations (the sound field). The large arrows in Figure 2-8 are the directions of fluid motion.

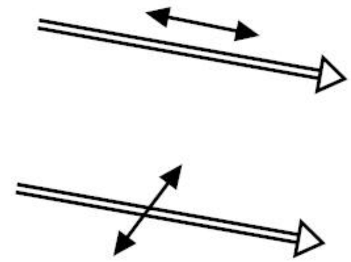

Fig. 2-8. Compression and shear body waves.

2.4.2 Transverse (Shear) Waves

These waves result in lateral oscillatory motion of the fluid. The main characteristic is that the material motion is in the plane and at right angles to the direction of the wave. It is associated with the viscous forces that create the motion. Each of the basic sound sources creates this type of wave but in most cases the magnitude of the motion is sufficiently small that it is neglected {2.3.7}.

2.5 Surface Wave Types

Surface waves can occur on elastic materials of any depth, such as on the surface of the earth, or on plates. Both have the potential to create sound in the adjacent medium.

2.5.1 Water Waves

These waves operate in liquids of finite but often great depth. They are waves that move along the surface and interact with the surrounding medium. Although they decay in magnitude with depth; the fluid motion is circular in a plane whose axis is at right angles to the direction wave travel, as seen in the first of Figure 2-9. They have a component of motion vertical to the surface.

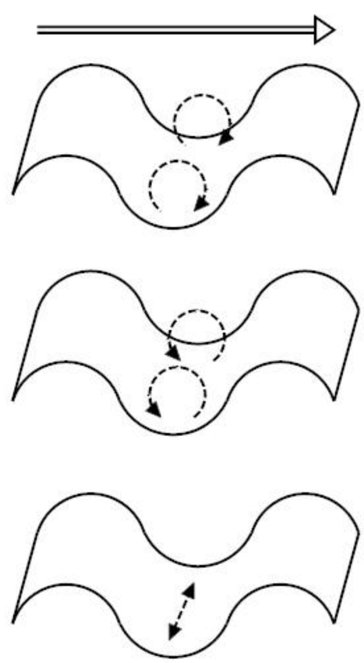

Fig. 2-9 Particle motion for three wave types.

2.5.2. Rayleigh Waves

These waves operate in materials of great depth. They are waves that move along the surface and interact with the surrounding medium. Although they decay in magnitude with depth, the material motion is retrograde elliptical in a plane whose axis is at right angles to the direction of wave travel, as seen in the second of Figure 2-9. They have a component of motion vertical to the surface also. The waves are named after J.W. Strutt (Baron Rayleigh 1842-1919).

2.5.3 Love Waves

These waves also operate in materials of great depth, and are waves that move along the surface and interact with the surrounding medium. Although they decay in magnitude with depth, material motion is lateral to the direction of the wave, as seen in the third of Figure 2-9. They are propagating shear waves and do *not* have a vertical component of motion as might be suggested by the simple diagram in the figure. The waves are named after A. E. H. Love (1863-1934).

2.5.4 Lamb Waves

These waves operate in materials of finite thickness. They are essentially internal longitudinal waves that cause the material to bulge and thin due to the Poisson effect causing surface waves that interact with the surrounding medium. The theory was developed by Horace Lamb (1849-1934).

2.5.5 Sound from Surface Waves

In each of the above cases, there is sinusoidal motion of the surface which is coupled to the surrounding medium. Does such motion create sound? There are many texts that address surface wave motion and the mathematics can get complex so only a general discussion will be given here. There are two cases: when the waves are traveling and when they are standing.

Standing waves occur when the surface is of finite length, such as a metal plate or supported membrane. All such motion has a prescribed frequency and node points between which the motion is finite. S*ound is created.* A simple model of standing waves is given in {3.11.4} and in the **SoundSource** program.

Traveling waves couple to the medium in different ways, depending on the surface wave speed. Morse and Ingard give a detailed mathematical analysis of this situation [1, p624]. When the surface wave speed $\mathbf{C_p}$ is less than that of sound in the medium $\mathbf{C_0}$, the medium motion is reactive and the periodic motion moves with the surface wave but decays exponentially in the direction perpendicular to the surface. *No sound is created.* When the wave speed is greater than the sound speed, *sound is created.* The waves move outward at the Mach angle $\cos\theta = c_0/c_p$. As the surface wave speed just passes sound speed the waves move along the surface as if a plane source was on one end of the surface. If the surface wave speed is much greater than the sound

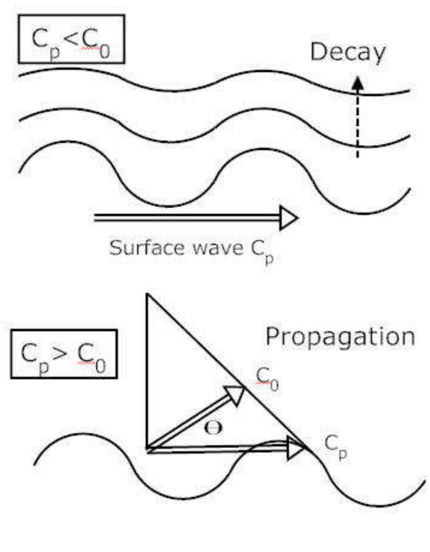

Fig. 2-10. Medium motion due to a surface wave.

speed, the waves move almost perpendicular to the surface, much like a plane surface moving perpendicular to itself.

Key Points: Standing surface waves will always create sound. Travelling surface waves can create sound when the surface wave speed exceeds that of sound in the medium. The radiation has a specific direction based on the ratio of the two speeds. The characteristic speed is that of the surface wave.